National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"

Department of
Mathematical Analysis
and Probability Theory

LINEAR ALGEBRA AND ANALYTIC GEOMETRY

Syllabus

Requisites of the Course

Level of higher education	***First (bachelor's)***
Field of Study	*12 Information technologies*
Specialty	*121 Software Engineering*
Educational program	*Computer Systems Software Engineering*
Status of the discipline	*Normative*
Form of study	*full-time*
Year of study, semester	*1st year, 1st semester*
Scope of the discipline	*4 ECTS credits / 120 hours*
Semester control/ control measures	*Final test/Modular control work*
Class schedule	*http://rozklad.kpi.ua*
Language of instruction	*English*
Information about the course leader / teachers	Lecturer: *Iryna Petrivna Blazhievska,PhD, i.blazhievska@gmail.com* Teachers of practical work*: Iryna Petrivna Blazhievska, PhD, i.blazhievska@gmail.com,* *Oleksandr M. Moklyachuk, PhD, omoklyachuck@gmail.com* *Tatiana Skorobohach, TetianaSkorobohach@gmail.com*
Placement of the course	Link to the remote resource in Moodle: https://do.ipo.kpi.ua/course/view.php?id=263

Program of the discipline

1. Description of the discipline, its purpose, subject matter and learning outcomes

The basics of linear algebra and analytic geometry, together with the course of higher mathematics, lay the foundation for a specialist's mathematical and engineering education, promote the development of logical thinking, and ensure the formation of a qualified creative specialist. The discipline is important in the training of computer engineering specialists, in particular because it is closely related to professional disciplines and helps to understand complex phenomena of mathematical, physical, and information nature.

Course Objectives

- To teach students to use linear algebra methods in engineering calculations;
- Learn to create geometric models of the objects under consideration;
- To develop the ability to bring the solution of the problem to a practically acceptableresult - a number, graph, surface, accurate qualitative conclusion using adequate computing tools, models, tables and reference books;
- Develop the ability to analyze the results obtained, independently use and study the literature on mathematics, and apply classical models in practice.

Subject of Discipline

The main sections of analytical geometry and linear algebra.

Learning outcomes

The discipline provides the following general and professional competencies: GC1, GC2, GC6, PC17, PLO19, PLO5, PLO11, PLO27.

2. Prerequisites and post-requisites of the discipline (place in the structural and logical scheme of study in the relevant educational program)

Prerequisites: School course in mathematics (algebra, geometry).

Post-requisites: Higher mathematics.

3. Content of the discipline

The discipline consists of the following topics:

Topic 1: Matrices, determinants, and systems of linear equations.

Topic 2. Vector algebra.

Topic 3: Analytical geometry on the plane and in space.

Topic 4. Linear spaces and linear operators. Quadratic forms.

4. Training materials and resources

Basic

1. Ordynska Z.P., Orlovskyi I.V., Runovska M.K. Analytical geometry and linear algebra: lecture notes for students of technical faculties. - K.: NTUU "KPI", 2014. - 176 p.
2. Alekseeva I. V., Gaidey V. O., Dykhovychnyi O. O., Fedorova L. B. Linear algebra and analytic geometry. Workshop. - K.: NTUU "KPI", 2011. - 184 c.
3. Linear algebra (matrices, determinants, ranks, systems of linear equations). Video workshop: https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=202796
4. Vector algebra (linear operations on vectors, scalar, vector, mixed products and their applications). Video workshop: https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=202797
5. Blazhiyevska I.P. Analytical geometry on the plane (construction, equations of curves). Video workshop: https://www.youtube.com/playlist?list=PLfABByLvmIIU9yjOZ5efHa5xH74VyszDU
6. Analytical geometry. Linear algebra: Collection of tasks for a typical calculation work for 1st year students of technical faculties / Compiled by: Konovalova N.R., Baranovska G.G. and others - K.: IVC "Polytechnic", 2001.

Auxiliary

7. Dubovyk V.P., Yurik I.I. Higher Mathematics / V.P. Dubovyk, I.I. Yurik - K.: Higher School, 1998.
8. Buldyhin V.V., Zhuk V.A., Rushchytska S.O., Yasinsky V.A. Collection of problems in analytical geometry and vector algebra / V.V. Buldyhin, V.A. Zhuk, S.O. Rushchytska, V.A. Yasinsky - K.: Vyshytsia Shkola, 1999. - 191 p.

Educational content

5. Methods of mastering the discipline (educational component)

Lectures

1. Matrices and actions on them.
   The concept of a matrix. Arithmetic operations on matrices: addition, multiplication of a matrix by a number, multiplication of matrices. Elementary transformations of matrices.
   Recommended reading: [3], c. 10-14.
2. Determinants and their properties.
   Determinants of the 2nd and 3rd orders, definitions and calculations. Minors. Algebraic complements. Calculating determinants of the nth order by decomposing the determinant by the elements of its row or column of the matrix. Properties of determinants. Recommended reading: [3], c. 14-17.

3. Inverse matrices.
   Matrix equations. The rank of the matrix. The concept of the inverse matrix. Matrix equations. Calculating the rank of a matrix.
   Recommended reading: [3], c. 18-21.
4. Systems of linear algebraic equations.
   Basic definitions. Kronecker-Capelli theorem. Methods for solving nondegenerate systems of linear algebraic equations.
   Recommended literature: [3], c. 22-26.
5. Gaussian method.
   Homogeneous systems of linear algebraic equations. Gauss method for solving arbitrary systems of linear algebraic equations. Fundamental system of solutions for homogeneous systems of linear algebraic equations.
   Recommended reading: [3], c. 26-30.
6. Vectors and actions on them.
   Vectors: basic definitions. Linear operations on vectors. Projection of a vector on an axis. Operations on vectors given by projections.
   Recommended reading: [3], c. 31-38.
7. Linear dependence and independence of a system of vectors.
   The basis of the system of vectors. Linear dependence and independence of a system of vectors. The basis of a system of vectors. Decomposition of a vector by its basis in the plane and in space.
   Recommended literature: [5], c. 46-50.
8. Scalar and vector products of vectors.
   Scalar product of vectors, its properties. Expression in terms of vector coordinates. Application of the scalar product. Vector product of vectors. Definition, geometric interpretation, properties.
   Recommended literature: [3], c. 38-43.
9. Vector and mixed product of vectors.
   Expression of a vector product in terms of coordinates. Application. Mixed product of vectors. Definition, geometric interpretation, properties, expression in terms of coordinates of factors, application.
   Recommended literature: [3], c. 43-47.
10. Coordinate systems on the plane.
    A line on the plane. Coordinate systems on the plane. A line on the plane, different types of its equation. Basic problems for a line on a plane.
    Recommended reading: [3], c. 48-52, 56-62.
11. Second-order curves in the plane: ellipse, hyperbola, parabola.
    Second-order curves in the plane: ellipse, hyperbola, parabola. Canonical equations, characteristics of second-order curves.
    Recommended reading: [3], c. 62-75.
12. A coordinate system in space.
    A plane in space. The coordinate system in space. Plane in space, different types of its equation. Basic problems for a plane in space.
    Recommended literature: [3], c. 76-81.
13. Straight in space.
    Problems on a line and a plane in space. A line in space, different types of its equation. The relative position of two lines in space. Problems on a line and a plane in space.
    Recommended reading: [3], c. 81-88.
14. Second-order surfaces.
    General equation of a second-order surface. Characteristics and shape of basic surfaces of the second order.

Recommended reading: [3], c. 88-96.

15. Linear spaces.
    Linear space: definition and examples. Dimension and basis of linear spaces.
    Recommended reading: [5], c. 42-51.
16. Line operators.
    Linear operators and their matrices. Transformation of the matrix of a linear operator when switching to a new basis. Eigenvalues and eigenvectors of a linear operator.
    Recommended literature: [5], c. 104-106, 111-118.
17. Euclidean spaces.
    Euclidean spaces: basic definitions. Orthonormalized basis of Euclidean space.
    Recommended reading: [5], c. 80-91.
18. Square shapes.
    The concept of a square shape. Reduction of the square form to the canonical form. Significant quadratic forms. Recommended reading: [5], c. 182-190.

- **Practical classes**

1. Matrices and actions on them. Determinants.
2. Inverse matrices. Matrix equations. Methods for solving non-degenerate systems of linear algebraic equations.
3. The rank of the matrix. Gauss method for solving arbitrary systems of linear algebraic equations. Linear homogeneous systems, their PDEs.
4. Vectors and actions on them. Linear dependence and independence of a system of vectors. The basis of a system of vectors.
5. Scalar, vector and mixed products of vectors, their application in geometry and physics. A line on the plane.
6. A plane in space. A line in space. The relationship between lines and planes in space.
7. Curves of the 2nd order (ellipse, hyperbola, parabola). Surfaces of the 2nd order.
8. Linear operators. Eigenvalues and eigenvectors of a linear operator. Quadratic forms.
9. Modular control work.

6. Self-study work of the student

Students' self-study work includes:

1. Completion of homework on the course topics;
2. Completion of home test work;
3. Preparing for practical classes;
4. Preparing and completing a module test.

Policy and control

7. Policy of the academic discipline (educational component)

Recommended learning methods: studying the main and auxiliary literature on the topics of lectures, solving problems in practical classes and doing homework.
Students are encouraged to keep detailed lecture notes. Independent work is an important aspect of the qualitative assimilation of the material, development of methods and algorithms for solving the main tasks of the discipline. It includes reading the literature, reviewing the literature on the topic, preparing for classes, completing homework, preparing for the MCQs and the test.

Academic integrity

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute". For more information: https://kpi.ua/code

Standards of ethical behavior

The norms of ethical behavior of students and employees are defined in Section 2 of the Code of Honor ofthe National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute". For more information: https://kpi.ua/code

8. Types of control and rating system for assessing learning outcomes (RSO)

Current control: work in practical classes, homework, writing control measures (part of the DCD).

- **Class participation (Uc)** is assessed by the instructor based on the student's activity in class and atthe board. A total of 5 points can be earned for class work per semester.
- **The completion of the home control work (Uh)**, subject to the successful and timely defense of individual sets of tasks, is evaluated at 15 points per semester (for all parts of the DCD).

Calendar control: checks the student's performance of the current set of tasks and control activities. To successfully pass the calendar control, a student must complete at least 50% of all tasks.

Module control: *The completion of the m*odular examination (Um) allows the student to score 30 points of the rating. For convenience, the module test can be divided into 3 thematic tests of 10 points each - and defended in parts (in pairs provided for consultations)**.**

Semester control: final test.

Conditions of admission to the semester control**:** A student has gained 30 rating points during the semester. Final test work (Ut) is held during the winter session and is evaluated at 50 rating points. The work consists of 5 questions - 1 theoretical question and 4 problems from Topics 1-4 of the discipline "Linear Algebra and Analytic Geometry" - 10 points each.

The semester rating consists of the points of current control (class work, DCD in parts), module control (3-part ICR) and examination work.

The semester rating scale is 100 points and is calculated as follows:

Rc = **Uc** + **Uh** + **Um** + **Ut** = 5 + 15 + 30 + 50 = 100 points.

9. Table of correspondence between rating points and grades on the university scale:

Number of points	*Assessment*
100-95	Excellent
94-85	Very good
84-75	Good
74-65	Satisfactory
64-60	Sufficient
Less than 60	Fall
The conditions for admission are not met	Not allowed

10. Additional information on the discipline (educational component)

The list of theoretical questions to be tested:

1. Matrices and actions on them (definitions, actions, transformations). Examples.
2. Determinants of the 2nd and 3rd orders (definitions, properties).
3. Inverse matrix (definition, properties, search).
4. Matrix equations (basic models).
5. Matrix rank (definitions, Gaussian and envelope minor methods).
6. SLAR (definition, matrix method for finding solutions to square nondegenerate matrices).
7. SLAR (definition, Kramer's method for finding solutions to square nondegenerate matrices).

8. Arbitrary SLARs (definition, Gaussian method of solving systems).
9. Homogeneous SLARs. A system of fundamental solutions.
10. Geometric vectors in the plane and space (definitions, linear operations).
11. Projection of a vector onto an axis. Decomposition by orths of coordinate axes. The modulus of the vector. The directional cosines.
12. Actions on vectors given by projections. Point and vector coordinates.
13. Vectors in an n-dimensional algebraic space (definition, actions). Examples.
14. Linear dependence and independence of a system of vectors. Examples.
15. Base and basis of a system of vectors. Examples.
16. Vector bases on the plane and in space. Examples.
17. Scalar product of vectors (definition, properties, applications).
18. Vector product of vectors (definition, properties, applications).
19. Mixed product of vectors (definition, properties, applications).
20. Coordinate systems on the plane. Transformations of the Cartesian coordinate system. Equation of a curve (line) on a plane.
21. Polar coordinate system (transition formulas, basic curves). Examples.
22. A line on a plane (different types of equations). Basic problems for a line on a plane.
23. Coordinate system in space. Equations of surface and line in space.
24. Plane in space (different types of equations).
25. Basic problems for a plane in space.
26. A line in space (different types of equations).
27. Basic problems on a line in space.
28. Basic line and plane problems in space.
29. 2nd order curves on the plane (general equation, classification according to parameters).
30. Ellipse, its canonical equation, construction and characteristics.
31. Hyperbola, its canonical equation, construction, and characteristics.
32. Parabola, its canonical equation, construction, and characteristics.
33. Surfaces of the 2nd order (general equation). Basic surfaces of the 2nd order (canonical equation, construction and characteristics).
34. Linear spaces. The basis and dimension of a linear space.
35. Relationship between bases in n-dimensional linear space. Transforming coordinates when changing the basis.
36. Euclidean space. The base in Euclidean space.
37. Orthonormalized basis (Gram-Schmidt orthogonalization method).
38. Orthogonal operator.
39. Square forms (definition). Reduction to the canonical form.
40. Square shapes (definition). Significant quadratic shapes. Sylvester's criterion.

Work program of the academic discipline (syllabus):

Designed by Associate Professor, PhD, Blazhievska Iryna Petrivna.

Adopted by the Department of Mathematics (protocol № 11 dated 04.06.2022)

Approved by The Methodological Commission of the FMF (protocol № 13 dated 22.06.2022.)

Approved by The FIOT Methodological Commission (protocol № 10 dated 10.06.2022.)