



# DISCRETE MATHEMATICS

(Syllabus)

## Реквізити навчальної дисципліни

<b>Level</b>	<i>First (bachelor)</i>
<b>Branch of knowledge</b>	12 Information technologies
<b>Specialty</b>	123 Computer engineering
<b>Educational program</b>	Computer systems and networks
<b>Discipline status</b>	<i>Normative</i>
<b>Form of education</b>	<i>Full-time, part-time</i>
<b>Year, semester</b>	<i>1-st year</i> , spring semester
<b>Scope of the discipline</b>	<i>4 credits, 120 hours</i>
<b>Semester control/ control measures</b>	<i>Test</i>
<b>Timetable</b>	Lectures - 36 hours. Laboratory work – 18 hours. Independent student works – 66 hours
<b>Language of teaching</b>	Ukrainian, English
<b>Information about the course leader / teachers</b>	Lecturer (full-time): D.Sc., professor, Mykhailo Anatoliyovych Novotarskyi Lecturer (part-time): assistant Artem Mykolayovych Ponomarenko <a href="mailto:novotar@gmail.com">novotar@gmail.com</a> , <a href="https://cutt.ly/JwHHzyly">https://cutt.ly/JwHHzyly</a> Laboratory: assistant Artem Mykolayovych Ponomarenko, <a href="mailto:ponomarenkokpi@gmail.com">ponomarenkokpi@gmail.com</a>
<b>Placement of the course</b>	<a href="https://classroom.google.com/c/NDQ3OTg3MTUwMTQw?cjc=s2vkqdx">https://classroom.google.com/c/NDQ3OTg3MTUwMTQw?cjc=s2vkqdx</a>

## Program

**1. Description of the educational discipline, its purpose, subject of study and learning outcomes** The discipline "Discrete Mathematics" is intended for a thorough study of modern methods of discrete mathematics, providing fundamental training in the field of the theory of discrete structures, which are the basis of computer engineering.

The educational discipline contains two main sections:

1. Theory of sets.
2. Graph theory.

Within the framework of the first section, the main provisions of set theory are considered, identities, the Cartesian product of sets, and the power of sets are studied. Considerable attention is paid to the study of relations on sets, properties of relations, images and prototypes, operations on relations. Binary relations and composition of binary relations are studied. Types of relations are considered: equivalence relations, relations of partial and full order.

The chapter also contains the basics of combinatorics. The main rules of combinatorics, methods of forming samples from a set of set elements, standard combinatorial samples are considered. Particular attention is paid to algorithms for generating subsets that correspond to standard combinatorial sampling. The second section includes issues of graph theory. In this section, the basic concepts of graph theory, set-theoretic properties of graphs, methods of specifying graphs and operations on graphs are studied. Isomorphism of graphs, set-theoretic operations on graphs, reachability and connectivity of graphs, independent cycles of a graph, cyclomatic number of a graph are considered. The planarity of graphs, the generation of planar graphs, the planar graph construction algorithm, and the planarity check are considered separately. In this meaningful module, the fundamental set of graph cycles is studied, finding

cycles using the depth-first search algorithm, cycle matrices and the cut of directed graphs, Euler graphs, directed Euler graphs, the algorithm for constructing an Euler cycle, Hamiltonian graphs.

**The purpose** of studying the course "Discrete Mathematics" is a thorough study of modern methods of discrete mathematics, fundamental training of students in the field of the theory of discrete structures, which are the basis of computer engineering, promotion of the development of logical and analytical thinking of students.

**The main tasks** of the academic discipline.

According to the requirements of the educational-professional program, after mastering the academic discipline, students must demonstrate the following learning outcomes:

**Knowledge:** basic concepts of set theory, combinatorics and graph theory; basic formulas and methods of combinatorics; algorithms and means of finding optimal solutions to typical problems of discrete mathematics.

**Ability:** to have the basics of the theory of sets and relations, to formulate and solve problems related to the use of graphs, to perform formulation and solution of problems of synthesis and analysis of discrete objects; to find the most effective mathematical apparatus for solving a specific problem.

**Experience:** the student must know the basic principles of developing combinatorial algorithms, algorithms

that describe relations on sets, algorithms on graphs, investigate the properties of these algorithms, identify their advantages and disadvantages, choose optimal algorithms for solving the given problem, perform analysis and processing of the results of solutions solving problems on sets and graphs, using optimization methods.

**Basic competences**

After mastering the normative discipline "Programming. Part 1", bachelor's degree holders must acquire the following:

**general competencies:**

**GC1.** Ability to abstract thinking, analysis and synthesis.

**GC3.** Ability to apply knowledge in practical situations.

**professional competences:**

**PC12.** The ability to identify, classify and describe the operation of software and technical tools, computer and cyber-physical systems, networks and their components by using analytical methods and modeling methods.

**Main program learning outcomes**

**PRO1.** Know and understand the scientific principles underlying the functioning of computer tools, systems and networks

**PRO7.** To be able to solve problems of analysis and synthesis of means characteristic of the specialty.

**PRO13.** Be able to identify, classify and describe the operation of computer systems and their components

**2.Pre-requisites and post-requisites of the discipline (place in the structural-logical scheme of training according to the relevant educational program)**

The educational base of the course is the amount of mathematical knowledge that meets the requirements of the "Higher Mathematics" course. Course materials are used in the disciplines "Probability Theory and Mathematical Statistics", "Computer Logic", "Algorithms and Calculation Methods", "Computer Modeling", "Computer Network Design", "Organization of Databases", "Fundamentals of software engineering", "System programming" and others. The material of the educational discipline is related to the materials studied in the courses "Higher Mathematics: Differential Calculus, Linear Algebra", "Programming" and "Discrete Mathematics". Knowledge and practical skills acquired within the framework of this educational discipline can be applied, when studying the following courses: "Computer modeling", "Fundamentals of software engineering", "System programming", "Technology of distributed computing", "Network and information technologies" and other.

### **3.Content of the academic discipline**

The discipline "Discrete Mathematics" includes the study of the following topics.

#### **Section 1. Theory of sets**

##### **Topic 1.1. Basic provisions of set theory**

Basic definitions. Operations on sets. Venn diagrams. Identities of the algebra of sets. Splitting sets.

Covering sets. An ordered set or tuple. Algorithm for ordering a set. Cartesian product of sets. Designing.

### **Topic 1.2. Correspondence and relationship**

Conformity. Basic concepts. Types of matches. The concept of relation. Definition of relation. Scope and set of values. Methods of specifying binary relations. Section of the relation through the element. Operations on relations. Additional operations (inverse relation, composition of relations). Special properties of relations.

### **Topic 1.3. Equivalence relation**

Definition of equivalence ratio. Properties of equivalence relations. Equivalence classes.

### **Topic 1.4. Order relation**

Examples of order relationships. Determination of order relations. Terminology and notation. Types of order relations. Basic concepts of ordered sets. Linearly ordered sets. Properties of linearly ordered sets. A completely ordered set. A partially ordered set. Partitioning of a partially ordered set on a chain. Definition of the largest element of a set. Definition of the maximum element of a set. Determination of the smallest and minimum elements of a set. Determination of the upper and lower faces of the set. Hasse diagrams. An example of creating non-contradictory relations S and R.

### **Topic 1.5. Functions and their properties**

Definition of a function. Definition of reflections, their properties and types. Methods of specifying functions. Special functions. A function of two variables. Matrices, operations on matrices. The concept of an functional. The concept of an operator.

### **Topic 1.6. Introduction to combinatorics**

Basic concepts of combinatorics. Basic rules of combinatorics. Placement with repetitions. Placement without repetitions. Permutations without repetitions. Permutations with repetitions. Compounds (combinations) without repetitions. Compounds (combinations) with repetitions. Splitting the set into subsets. Identities for compounds.

### **Topic 1.7. Basic combinatorial algorithms**

Subset generation algorithms. Generating all subsets. Algorithm for generating all binary vectors of length

n in lexicographic order. Generating subsets with a condition. Generation of k-element subsets.

Algorithms

of permutations. Selection by sorting.

## **Section 2. Graph theory**

### **Topic 2.1. Basic provisions of graph theory**

The history of graph theory. Basic definitions of graphs. Contiguity. The degree of the top. Theorems about degrees of graph vertices. Graphs with constant and variable degrees of vertices. Subgraph Circulating graphs. Structural characteristics of graphs. Graph connectivity. Plural of cut, cut and bridge.

### **Topic 2.2. Methods of setting and properties of graphs.**

Operations with elements of graphs. Assignment of graphs in mathematics. Isomorphism of graphs. Graph isomorphism recognition algorithm. Theoretical multiple operations on graphs. Pairing of the edges of the graph.

### **Topic 2.3. Relations and display on graphs**

Graphs and binary relations. Connection between operations on graphs and operations on relations. Multivalued mappings. Representation of a set of vertices. Definition of a graph and its properties using mappings. Reachability and counterreachability of a vertex in graphs.

### **Topic 2.4. Count numbers**

Cyclomatic number. Internal stability number. External stability number.

### **Topic 2.5. Trees and their properties, forest, cycles**

Definition of a tree, properties of trees. Procedures for building a framework tree and forest. Properties of cyclic rank. A fundamental system of graph cycles. The skeleton of the least weight. Kruskal's algorithm. Prim's algorithm.

### **Topic 2.6. Traversal of graphs. Substantive provisions**

Detour in depth. A graph depth traversal program. Bypass in width. Program for traversing a graph in width.

### Topic 2.7. Algorithms for finding the shortest paths in a graph

Searching for paths in a graph by Terry's algorithm. Wave algorithm. Finding the shortest path in a weighted graph using Dijkstra's algorithm. Ford-Bellman algorithm for finding the minimum path. Algorithm of Floyd-Warshell.

### Topic 2.8. Graph coloring

Coloring tasks. Basic definitions. Chromatic number. Chromatic number and standard characteristics. Chromatic number and graph density. The three lower grades of the chromatic number. The upper estimate of the chromatic number. Brooks' theorem. Theorems about six, five and four colors.

### Topic 2.9. Basic graph coloring algorithms

Basic information. Sequential coloring algorithm. Implicit sorting algorithm. An example of an implicit sorting algorithm. Recursive sequential coloring procedure. An example of a recursive procedure. "Greedy" coloring algorithm. Example of work "greedy" coloring algorithm. Results of sequential coloring algorithms. Heuristic coloring algorithm. An example of a heuristic coloring algorithm. Modified heuristic coloring algorithm. An example of a modified heuristic coloring algorithm. Graph coloring by A.P. method. Yershova. An example of graph coloring by the A.P. method.

### Topic 2.10. Euler's paths and cycles. Flat and planar graphs

Euler's paths and cycles. Hamilton cycles (basic definitions). Flat and planar graphs. General concepts of a planar graph. Non-planar graphs. Faces of a planar graph. Euler's theorem. Homeomorphic graphs. The Pontryagin-Kuratovsky theorem. Debit operation. Wagner's theorem.

### Topic 2.11. Networks. Construction of a network of minimum length. Flows in networks, maximum flow in the network, redistribution of flow.

Transport networks. The network bandwidth search algorithm. Flows in networks, maximum flow in the network, redistribution of flow. Ford-Falkerson algorithm. Network planning. Analytical evaluation of the network.

## 4. Educational materials and resources

1. Levin O. Discrete Mathematics: An Open Introduction 3<sup>rd</sup> Edition  
<https://discrete.openmathbooks.org/pdfs/dmoi3-tablet.pdf> for free at  
<http://discrete.openmathbooks.org/>
2. Susanna S. Discrete Mathematics with Applications 4th Edition 2016, - 984 p.

## Навчальний контент

### 5. Methods of mastering an educational discipline (educational component)

The structure of the discipline "Discrete Mathematics" is presented in Table 1.

Table 1

The structure of the discipline "Discrete Mathematics"

Topics	Hours			
	Total	Including		
		Lectures	Labs	ISW
<b>Section 1. Basics of set theory</b>				
<b>Topic 1.1. Basic provisions of set theory</b> Basic definitions. Operations on sets. Venn diagrams. Identities of the algebra of sets. Splitting sets. Covering sets. An ordered set or tuple. Algorithm for ordering a set. Cartesian product of sets. Designing.	6	3	2	1
<b>Topic 1.2. Correspondence and relatio</b>	6	2	0	4

Topics	Hours			
	Total	Including		
		Lectures	Labs	ISW
Correspondence. Basic concepts. Types of matches. The concept of relation. Definition relation. Scope and set of values. Methods of specifying binary relations. Section of the relation through the element. Operations on relations. Additional operations (inverse relation, composition of relations). Special properties of relations.				
<b>Topic 1.3. Equivalence relation</b> Definition of equivalence relation. Properties of equivalence relations. Classes of equivalence.	10	2	2	6
<b>Topic 1.4. Order relation</b> Examples of order relationships. Determination of order relations. Terminology and notation. Types of order relations. Basic concepts of ordered sets. Linearly ordered sets. A completely ordered set. A partially ordered set. Partitioning of a partially ordered set on a chain. Definition of the largest element of a set, the maximum element of a set. Determination of the smallest and minimum elements of a set. Determination of the upper and lower faces of the set. Hasse diagrams. An example of creating non-contradictory relations S and R.	10	2	2	6
<b>Topic 1.5. Functions and their properties</b> Definition of a function. Definition of reflections, their properties and types. Methods of specifying functions. Special functions. A function of two variables. Matrices, operations on matrices. The concept of an operator.	4	2	0	2
<b>Topic 1.6. Introduction to combinatorics</b> Basic concepts of combinatorics. Basic rules of combinatorics. Placing. Permutations. Compounds Splitting the set into subsets. Identities for compounds.	7	2	0	5
<b>Topic 1.7. Basic combinatorial algorithms</b> Subset generation algorithms. Generating all subsets. Algorithm for generating all binary vectors of length n in lexicographic order. Generating subsets with a condition. Generation of k-element subsets. Generation of k-element subsets. Algorithms of permutations. Selection by sorting...	10	2	2	6
Control work 1	1		0	1
<b>Section 2. Graph theory</b>			0	

Topics	Hours			
	Total	Including		
		Lectures	Labs	ISW
<b>Topic 2.1. The main provisions of graph theory.</b> The history of the emergence of graph theory. Basic definitions of graphs. Contiguity. The degree of the top. Theorems about degrees of graph vertices. Graphs with constant and variable degrees of vertices. Subgraph Circulating graphs. Structural characteristics of graphs. Graph connectivity. Plural of cut, cut and bridge.	3	2	0	1
<b>Topic 2.2. Methods of setting and properties of graphs</b> Operations with elements of graphs. Assignment of graphs in mathematics. Isomorphism of graphs. Graph isomorphism recognition algorithm. Theoretical multiple operations on graphs. Pairing of the edges of the graph.	4	2	0	2
<b>Topic 2.3. Relations and mappings on graphs</b> Graphs and binary relations. Connection between operations on graphs and operations on relations. Multivalued mappings. Representation of a set of vertices. Definition of a graph and its properties using mappings. Reach and counterreach vertices in graphs.	4	2	0	2
<b>Topic 2.4. Graph numbers</b> Cyclomatic number. The number of internal stability External stability number.	2	1	0	1
<b>Topic 2.5. Trees and their properties, forest, cycles</b> Definition of a tree, properties of trees. Procedures for building a framework tree and forest. Properties of cyclic rank. A fundamental system of graph cycles. The skeleton of the least weight. Kruskal's algorithm. Prim's algorithm	4	2	0	2
<b>Topic 2.6. Traversal of graphs. Substantive provisions</b> Basic provisions In-depth detour. A graph depth traversal program. Bypass in width. Program of the circulator in width.	3	2	0	1
<b>Topic 2.7. Algorithms for finding the shortest paths in a graph</b> Searching for paths in a graph by Terry's algorithm. Wave algorithm. Finding the shortest path in a weighted graph using Dijkstra's algorithm. Ford-Bellman algorithm for finding the minimum path. Algorithm of Floyd-Warshell.	10	2	2	6
<b>Topic 2.8. Graph coloring</b> Coloring tasks. Chromatic number. Chromatic number and standard characteristics. Chromatic	10	2	4	4

Topics	Hours			
	Total	Including		
		Lectures	Labs	ISW
number and graph density. Estimates of the chromatic number. Brooks' theorem. Theorems about six, five and four colors. Problem distribution of equipment. The task of making a schedule.				
<b>Topic 2.9. Basic graph coloring algorithms</b> Basic information. Sequential coloring algorithm. Implicit sorting algorithm. Recursive sequential coloring procedure."Greedy" coloring algorithm. Results of sequential coloring algorithms. Heuristic coloring algorithm. Modified heuristic coloring algorithm. Graph coloring by A.P. method. Yershova.	10	2	4	4
<b>Topic 2.10. Euler's paths and cycles. Flat and planar graphs</b> Euler's paths and cycles. Hamilton cycles. Flat and planar graphs. Homeomorphic graphs. Theorem Pontryagin-Kuratovsky. Debit operation. Wagner's theorem.	6	2	0	4
<b>Topic 2.11. Networks. Construction of a network of minimum length. Flows in networks, maximum flow in the network, redistribution of flow.</b> Transport networks. The network bandwidth search algorithm. Flows in networks, maximum flow in the network, redistribution of flow. Ford-Falkerson algorithm. Network planning. Analytical evaluation of the network.	3	2		1
Control work 2	1		0	1
<b>Test</b>	<b>6</b>		<b>0</b>	<b>6</b>
<b>Total in semester:</b>	<b>120</b>	<b>36</b>	<b>18</b>	<b>66</b>

The topic of the lectures, that are formed in accordance with the topics considered within the academic discipline are listed in the Table 2.

Table 2

Lecture classes

No	The name of the topic of the lecture and a list of the main questions (a list of didactic tools, references to the literature and tasks on the ISW)
1	<b>Basic provisions of set theory.</b> Basic definitions. Operations on sets. Venn diagrams. Identities of the algebra of sets. <b>Tasks on ISW.</b> Basic laws of algebra of sets, proof of identity to program implementation, algorithm of inclusion and exclusion of set elements.
2	<b>Identities of the algebra of sets.</b> Splitting sets. Covering sets. An ordered set or tuple. Algorithm for ordering a set. Cartesian product of sets. Designing. Conformity. Basic concepts. Types of matches. One-to-one correspondence. One-to-many matching.

№	The name of the topic of the lecture and a list of the main questions (a list of didactic tools, references to the literature and tasks on the ISW)
	<p>Multiple-unique matching. Many-to-many matching.</p> <p><b>Tasks on ISW.</b> Specifying elements of sets in Python: generating subsets of a set: variables and constants, lists. Use of sequence type data: lists, tuples, dictionaries. Algorithms for determining the Cartesian product of sets and the power of sets .</p>
3	<p><b>Plural relations.</b> The concept of relation. Definition of relation. Scope and set of values. Methods of specifying binary relations (enumeration, predicate, graph, matrix). Section of the relation through the element. Operations on relations (Union, intersection, difference, addition. Operations union and intersection of arbitrary families of relations). Additional operations (inverse relation, composition of relations).</p> <p><b>Tasks on ISW.</b> Correspondence and relation: tabular and graphical representation. Algorithms of operations on binary relations.</p>
4	<p><b>Special properties of relations</b> (reflexivity, antireflexivity, symmetry, antisymmetry, asymmetry, transitivity, antitransitivity).</p> <p><b>Equivalent relation.</b> Definition of equivalence ratio. Properties of equivalence relations. Equivalence classes.</p> <p><b>Tasks on ISW.</b> Creation of an algorithm for setting equivalent relations.</p>
5	<p>Order relation. Examples of order relationships. Determination of order relations (strict, non-strict order). Terminology and notation. Types of order relations (strict full, strict partial, non-strict full, non-strict partial). Basic concepts of ordered sets. Linearly ordered sets. Properties of linearly ordered sets. A completely ordered set. A partially ordered set. Partitioning of a partially ordered set on a chain. Definition of the largest element of a set. Definition of the maximum element of a set. Determination of the smallest and minimum elements of a set. Determination of the upper and lower faces of the set. Hasse diagrams. An example of creating non-contradictory relations S and R.</p> <p><b>Tasks on ISW.</b> Finding majorants and minorants of a set, algorithms.</p>
6	<p><b>Functions and their properties.</b> Definition of a function. The domain of definition and the domain of function values. The image of the set and the element of the set, the prototype of the set and the element of the set. Definition of reflections, their properties and types (surjective, injective, bijective). Methods of specifying functions (tabular, analytical, graphical). Special functions (identity function, round down, round up, factorial, binary operation). Sequence. A function of two variables. Matrices, operations on matrices. The concept was functional. The concept of an operator.</p> <p><b>Tasks on ISW.</b> Performance of practical tasks on matrix operations. Algorithms for the formation of relations.</p>
7	<p><b>Introduction to combinatorics.</b> Basic concepts of combinatorics. Basic rules of combinatorics (Rule of sum. Rule of product. Rule of inclusions and exclusions). Placement with repetitions. Placement without repetitions. Permutations without repetitions. Permutations with repetitions. Compounds (combinations) without repetitions. Compounds (combinations) with repetitions. Splitting the set into subsets. Identities for compounds.</p> <p><b>Tasks on ISW.</b> Combinatorial algorithm of binary relations.</p>
8	<p><b>Basic combinatorial algorithms.</b> Subset generation algorithms. Generating all subsets. Algorithm for generating all binary vectors of length n in lexicographic order. Generating subsets with a condition. Generation of k-element subsets. Algorithms of permutations. Selection by sorting. Bubble sort, quick sort, merge sort.</p> <p><b>Tasks on ISW.</b> Sorting algorithms, checking the order and place of an element in a sequence. Relational set model.</p>
9	<p><b>Basic provisions of graph theory.</b> The history of graph theory. Basic definitions of graphs (undirected, directed, labeled, graph with loops, multigraph, hypergraph, complete, bipartite, complete bipartite graph). Contiguity. The degree of the top. Theorems about</p>



№	The name of the topic of the lecture and a list of the main questions (a list of didactic tools, references to the literature and tasks on the ISW)
	degrees of graph vertices. Graphs with constant and variable degrees of vertices. Subgraph Circulating graphs. Structural characteristics of graphs. Graph connectivity. Plural of cut, cut and bridge. <b>Tasks on ISW.</b> Analytical description of graphs and operations on graphs.
10	Methods of setting and properties of graphs. Operations with elements of graphs. Specifying graphs in mathematics (analytical method: explicitly, by predicate, recursive procedure; graphical method; matrix method: by incidence matrix and adjacency matrix; specifying a graph using a list of edges). Properties of the incidence matrix of an undirected graph. Properties of the incidence matrix of a directed graph. Properties of the adjacency matrix of a directed graph. Isomorphism of graphs. Property of adjacency matrices of isomorphic graphs. Graph isomorphism recognition algorithm. Theoretical multiple operations on graphs. Pairing of the edges of the graph. <b>Tasks on ISW.</b> Specifying graphs using data structures in the Python scripting language: operations on graphs. Adjacency and incidence lists.
11	<b>Relations and display on graphs.</b> Graphs and binary relations. Connection between operations on graphs and operations on relations. Multivalued mappings. Representation of a set of vertices. Definition of a graph and its properties using mappings. Reachability and connectivity of graphs. Reachability and counterreachability of a vertex in graphs. Graph numbers. Cyclomatic number. Number of internal stability. Number of external stability. <b>Tasks on ISW.</b> Construction of equivalence relation on graphs.
12	Trees and their properties, forest, cycles. Definition of a tree, properties of trees. Procedures for building a framework tree and forest. Properties of cyclic rank. A fundamental system of graph cycles. The skeleton of the least weight. Kruskal's algorithm. Prim's algorithm. Traversal of graphs. Substantive provisions. Detour in depth. A graph depth traversal program. Bypass in width. Program for traversing a graph in width. <b>Tasks on ISW.</b> Definition of properties of graphs: eccentricity, diameter and radius, reachability and regularity. Algorithms for finding graph cycles, cycle matrices and graph cuts.
13	<b>Algorithms for finding the shortest paths in a graph.</b> Searching for paths in a graph by Terry's algorithm. Finding a path in a graph using Terry's algorithm manually. Wave algorithm. Finding the shortest path in a weighted graph using Dijkstra's algorithm. Finding the shortest path in a weighted graph using Dijkstra's algorithm manually. Ford-Bellman algorithm for finding the minimum path. Algorithm of Floyd-Warshell. <b>Tasks on ISW.</b> Comparison of algorithms for finding the shortest paths in a graph.
14	Graph coloring. Coloring tasks. Chromatic number. Chromatic number and standard characteristics. Chromatic number and graph density. Estimates of the chromatic number. Brooks' theorem. Theorems about six, five and four colors. The problem of equipment distribution. The task of making a schedule. <b>Tasks on ISW.</b> Coloring tasks. The task of making a schedule.
15	Basic graph coloring algorithms. Basic information. Sequential coloring algorithm. Algorithm of direct implicit enumeration. An example of an implicit sorting algorithm. Recursive sequential coloring procedure. An example of a recursive procedure. "Greedy" coloring algorithm. Example of work "greedy" coloring algorithm. Results of sequential coloring algorithms. <b>Tasks on ISW.</b> "Greedy" coloring algorithm.
16	<b>Heuristic coloring algorithm.</b> An example of a heuristic coloring algorithm. Modified heuristic coloring algorithm. An example of a modified heuristic coloring algorithm. Graph

№	The name of the topic of the lecture and a list of the main questions (a list of didactic tools, references to the literature and tasks on the ISW)
	coloring by A.P. method. Yershova. An example of graph coloring by the Yershov method. <b>Tasks on ISW.</b> Heuristic coloring algorithm. Comparison of "greedy" and heuristic graph coloring algorithms.
17	<b>Euler's paths and cycles.</b> Flat and planar graphs. Euler's paths and cycles. Hamilton cycles. Flat and planar graphs. Homeomorphic graphs. The Pontryagin-Kuratovsky theorem. Debit operation. Wagner's theorem. <b>Tasks on ISW.</b> Algorithms for constructing and checking the planarity of graphs. Algorithms for constructing Euler and Hamilton graphs.
18	<b>Networks. Construction of a network of minimum length. Flows in networks, maximum flow in the network, redistribution of flow.</b> Transport networks. The network bandwidth search algorithm. Flows in networks, maximum flow in a network, redistribution of flow. Marking algorithm. Gomori-Hu algorithm. Ford-Falkerson algorithm. Network planning. Analytical evaluation of the network. <b>Tasks on ISW.</b> Algorithms for building networks, building networks of minimum and maximum weight, estimation of flow in the network.

## 6. Independent student work (ISW)

A student's independent work consists of a theoretical and a practical component. The theoretical component involves the study of additional material that deepens the knowledge obtained at the lecture. The material that needs to be further studied and literary sources for studying this material are listed in Table 2. The practical component of the student's independent work consists in performing laboratory work, the list of which is given in Table 3.

The purpose of conducting a cycle of laboratory work is for students to acquire the necessary practical skills in the use of discrete mathematics methods and the development of effective algorithms for their implementation in solving problems.

Таблиця 3

Перелік лабораторних робіт

№ з/п	Назва лабораторної роботи	Кількість ауд. годин
1	Laboratory work No. 1. Sets: basic properties and operations on them, Venn diagrams. (Chapter 1, Topic 1.1, Topic 1.2)	2
2	Laboratory work No. 2. Binary relations and their main properties, operations on relations. (Chapter 1, Topic 1.3, Topic 1.4)	4
3	Laboratory work No. 3. Matrix methods of representing graphs. Finding the shortest spanning trees of a graph. Finding the shortest paths in a graph. (Chapter 2, Topic 2.1, Topic 2.2, Topic 2.5, Topic 2.7)	4
4	Laboratory work No. 4. Combinatorics: permutations, placement, combination. (Chapter 1, Topic 1.6, Topic 1.7)	4
5	Laboratory work No. 5. Graph coloring. (Chapter 2, Topic 2.8, Topic 2.9)	4

## Policy and testing

### 7. Policy of academic discipline (educational component)

During classes in the discipline "Discrete Mathematics", students must follow certain disciplinary rules:

- it is forbidden to be late for classes;

- at the teacher's entrance, as a sign of greeting, persons studying at KPI named after Igor Sikorsky, must stand up;
- extraneous conversations or other noise that interferes with classes are not allowed;
- leaving the classroom during the lesson is allowed only with the teacher's permission;
- the use of mobile phones and other technical means is not allowed without the teacher's permission.

Laboratory works are submitted in person with a preliminary check of theoretical knowledge, which is necessary for the performance of laboratory work. Validation of practical results includes code review and execution of test tasks.

In the course of training, the teacher has the right to award up to 5 incentive points for early completion of laboratory work, for a demonstrated creative approach when completing an individual task, or for active participation in the discussion of issues related to the subject of a lecture or practical session.

The teacher may assign up to 5 penalty points for performing and submitting laboratory work after the specified deadline, for a significant number of missed classes, or for violating the rules of behavior in classes.

When conducting control measures and performing laboratory work, students must adhere to the rules of academic integrity. If a significant percentage of plagiarism or plagiarism is detected, the teacher may refuse to accept the given work and demand the honest implementation of the curriculum.

## **8.Types of control and rating system for evaluating learning outcomes (RSE)**

Types of control in the discipline "Discrete Mathematics" include:

### **Laboratory work**

Independent performance of 4 laboratory works + 1 bonus laboratory work is planned.

The topics of laboratory works are coordinated in time and content with the topics of lectures. Performing laboratory work in its entirety allows you to acquire practical skills in the application of methods for working with sets and graphs, as well as to master modern technologies for programming algorithms that are built on the basis of these methods.

### **Current control**

There are 5 current tests with closed tests in the TCEXAM system, which fully cover the subject of this academic discipline. Each current closed test contains 10 questions and lasts 10 minutes. The total time is 2 hours and includes testing time and time to solve organizational issues. In the case of distance learning, a closed current test is conducted at the beginning of the lecture that follows the lecture that concludes the next topic. In the case of face-to-face training, the time of the next current test is set by the teacher in agreement with the students.

### **Semester control**

The semester closed test consists of two attestation tests by sections of the academic discipline. The first certification test on the topics of Chapter 1 contains 30 questions and lasts 35 minutes.

The second certification test on the topics of Chapter 2 contains 40 questions and lasts 45 minutes.

### **Test**

A necessary condition for a student's admission to credit is his individual semester rating of not less than 30 points, the absence of full arrears from laboratory work and not less than one positive certification. If at least one of the mentioned requirements is not fulfilled, the student will not be admitted to the credit.

Assessment is conducted in the form of an interview with the student to objectively determine the level of knowledge, skills and practical skills acquired during the semester. Provided that the student receives a semester rating of more than 60 points, the credit can be issued automatically in the last lesson of this course.

Since the credit module has a semester certification in the form of a credit, the rating system is based on the RSE-1 type.

Assessment of individual types of student's academic work

Section 1		Section 2	
Modes	Max points	Modes	Max points
Implementation and defense of laboratory work No. 1 "Sets: basic properties and operations on them, Venn diagrams"	10	Implementation and defense of laboratory work No. 3 "Counts. Ways of presenting graphs. Root trees. Finding the shortest paths"	10
Implementation and protection of laboratory work No. 2 "Binary relations and their main properties, operations on relations"	10	Implementation and defense of laboratory work No. 4 "Graph coloring, coloring algorithms"	10
Current test #1. Sets	5	Current test #3. Tree Counts.	5
Current test #2. Relation	5	Current test #4. Coloring	5
		Current test #5. Combinatorics	5
<i>Total 1</i>	30	<i>Total 2</i>	35
Final test			35
<b>Semester rating (<math>R_{sem}</math>)</b>			<b>100</b>

The student's individual semester rating  $R_{sem}$  from the credit module consists of the points he receives for:

- 1) Semester test 35 points)
- 2) Performing laboratory work (40 points)
- 3) Current tests (5x5=25 points)

During the semester, students perform 4 laboratory works. Lab #5 is a bonus lab worth 5 points

The maximum number of points for each laboratory work: 10.

Points are awarded for:

- timeliness of submission of work for defense – 0-1 point,
- proper preparation of the protocol of laboratory work - 0-2 points,
- proper performance of meaningful task for work - 0-3 points,
- answers to theoretical questions of the teacher - 0-5 points.

Total for laboratory work (maximum number of points) – 40 (for section 1 – 20 points, for section 2 – 20 points).

Calculation of the size of the rating scale:

The sum of the weighted points of control measures during the semester is:

$R_{nomoch} = \sum_k r_k$ , where  $r_k$  is the maximum rating score of each of the control measures (current tests, laboratory works).

The semester rating consists of the current rating and the grade for the semester test:

$$R_{sem} = R_{nomoch} + ST = 65 + 35 = 100$$

The student's individual semester rating (final semester rating) is the sum of points received by the student during the semester by participating in the prescribed control measures (attestation tests, current tests and laboratory works).

A necessary condition for a student's admission to the credit is his individual semester rating of not less than 30% of , i.e. 30 points, 4 laboratory tests passed and one positive certification in the semester. If at least one of the mentioned conditions is not fulfilled, the student will not be admitted to the credit.

Students who have fulfilled all admission requirements and have a semester rating of at least 60 points can receive a corresponding positive grade without additional tests.

Students who scored a credit module rating of 30 to 60 points during the semester have the option of rescheduling the credit twice in an additional session. The first revision is conducted in the form of a test in the TCEXAM system. After the first retake, students who scored more than 60% of correct answers receive a credit. An interview with the teacher takes place at the second rescheduling. Credit can be obtained based on the results of the interview, provided that 60% of the answers to the teacher's questions are correct.

Computer testing. An equal number of questions are randomly selected from each topic. Semester test 35 points. The test is given time at the rate of 1 minute per question. The number of points for the test is equal to the percentage of correct answers. The current test is given time at the rate of 1 minute per question.

The semester rating grade is converted into a grade on the national scale and the ECTS scale in accordance with the table (Table 5).

Table 5

Correspondence of rating points to grades on the university scale

<i>Number of points</i>	<i>Mark</i>
100-95	Excellent
94-85	Very good
84-75	Good
74-65	Satisfactory
64-60	Enough
Менше 60	Unsatisfactory
Не виконані умови допуску	Not admitted

### **9. Additional information on the discipline (educational component)**

Teaching the discipline "Discrete Mathematics" for the specialty "Computer Engineering" has its own specificity, which is connected with the fact that the development and operation of computer equipment requires a detailed acquaintance with complex systems. Key elements of these systems are subject to description and analysis using methods of discrete mathematics. The basic concepts of set theory, combinatorics and the basics of graph theory should be presented taking into account the specifics of special disciplines to facilitate their successful assimilation in further education.

The goal of revitalizing the educational process is to encourage students to engage in educational and cognitive work and independent work. When teaching educational material, the use of such technologies as problem-based lectures, mini-lectures, and brainstorming is envisaged.

Problem-based lectures are aimed at the development of logical thinking, the involvement of students in independently solving the relevant problem both in the discipline "Discrete Mathematics" and in other disciplines related to preparation for specialties.

Mini-lectures involve the teaching of educational material in a short period of time and are characterized by a significant capacity of targeted information. A more detailed study of the proposed material is left for independent processing.

The defense of laboratory works is accompanied by answers that highlight the solution of specific questions.

**Working program of the academic discipline (syllabus):**

**Compiled by Doctor of Technical Sciences, Prof. Mykhailo Anatoliyovych Novotarskyi**

**Approved by the OT department (protocol No. 13 dated 05.10.2023)**

**Agreed by the Methodical Commission of FIOT (protocol No. 11 dated 29.06.2023)**